Heat and Momentum Transfer in Laminar Flow: Helium Initially at Plasma Temperatures

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The use of plasma generators for chemical synthesis makes it necessary to cool gases rapidly so as to freeze the desired products. A gas may be cooled rapidly by directing its flow through a small diameter heat exchange system, as has been done in chemical synthesis work (*9*). The design of such a system requires a prediction of the rates of heat and momentum transfer at temperature differences in the range of 10,000°R. Since such data are not currently available, the present investigation is concerned with the simultaneous heat and momentum transfer which occur when a gas, in laminar flow, is cooled under extreme temperature gradients in a water-cooled, copper tube. It is also concerned with the corresponding analogies.

Experimental data on the simultaneous laminar transfer of heat and momentum at extreme temperatures is nearly nonexistant. Cholette and Kroll (see *12*) report on laminar heat transfer without the corresponding nonisothermal momentum transfer data. Talbot (*16*) reports on nonisothermal momentum transfer without the corresponding heat transfer data. The former was at moderate temperature differences (concerned with the heating of air by steam) and the latter at comparable temperature gradients. The work of Taylor and Kirchgessner (*17*) presents simultaneous heat and momentum transfer data. The number of points available are limited, because the experiments were concerned with turbulent rather than laminar flow. In addition temperatures were limited to 5,900°R., since the experiments involved heating of the flowing stream. Browning and Sebastian (*4*) reported laminar heat transfer data only for considerably lower temperature gradients (less than 3,000°R. entrance) and lower Graetz numbers. The corresponding nonisothermal momentum transfer data were not obtained.

A mathematical solution of the laminar heat conduction problem for pipe flow has been accomplished by Graetz and reported in McAdams (*12*). The solution assumes a uniform wall temperature and heat conduction in the radial direction only (two assumptions approached in the present investigation) and constant viscosity and thermal conductivity (conditions not approached). The solution also assumes a parabolic velocity distribution, a condition which is also approached but which, as Deissler (*7*) points out, can never be completely true as long as heat transfer is occurring. For instance the cooling of a gas results in cooler temperatures near the wall than at the center, which in turn produces lower viscosities near the wall than at the center. The result is a flattening of the parabolic velocity pattern. As a limit the flow would be rodlike and similar to that associated with turbulent flow. A laminar solution for such rodlike flow has been made by Drew and is also reported in McAdams (*12*).

For laminar flow and isothermal conditions the usual equation for predicting the friction factor and thus the momentum transfer is $f = 16/N_{Re}$. Little work appears to have been done to determine its continued validity under nonisothermal conditions.

As to analogies between heat and momentum transfer in laminar flow, there appears to be none formally proposed and little data from which to construct one. However if one acknowledges molecular transport to be the mechanism of transfer in laminar flow, an analogy can be derived from the definitions of the molecular diffusivities. Suitable definitions are offered in Foust et al. (*8*).

Laminar flow was defined as existing when the Reynolds number, based on fluid properties evaluated at the bulk temperature of the gas, was 2,100 or less. This is the normal definition, but in the heating or cooling of a fluid the normal velocity profile associated with laminar flow is distorted and could result in a transition to turbulence at some other value.

EXPERIMENTAL STUDY

Figure 1 shows a schematic diagram of the plasma generator used to provide the high-temperature gas for the study. Details of the various geometric configurations used are given in reference *18*. A novel gas flow, similar to that in a cyclone, was used. (A movie is available of the system in operation.*) A heat exchange system (Figure 2) was attached to the generator. This exchange system or test section is composed of three copper units each approximately 1 in. long, water cooled, and having an internal gas flow tube ⅛ in. in diameter. The inside diameter matched that of the anode section. The working gas was heated to temperatures of the order of 50,000°R. and higher in the plasma arc, cooled partially in the anode section, and entered the heat exchange system at temperatures of the order of 3,000° to 15,000°R. The heat transferred from the heated gas to the cooling water could be readily calculated from a knowledge of the water rate and its temperature rise.

The pressure loss across the exchange system could be determined by measuring the pressure at the inlet to the test section (shown at the extreme left in Figure 2). The difference between this pressure and atmospheric pressure was measured directly with a manometer.

Both the rate of heat transfer, as indicated by a rise in cooling water temperature, and the rate of momentum transfer, indicated by a change in manometer reading, could be varied by changing either the helium flow rate (0.213 to 1.263 lb./hr.) or the electrical energy input (up to 45,000 B.t.u./hr.) to the arc.

PRELIMINARY CONSIDERATIONS

Since this study is concerned with laminar flow conditions, it was first necessary to determine whether the flow actually was laminar. If DG/μ must be less than 2,100, it is only necessary to find some means of evaluating the viscosity, since the tube diameter and the mass velocity could be measured directly. Closely associated with the viscosity estimation is the value of the thermal conductivity (to be used in the Nusselt number). Consequently these estimated properties will be discussed together.

Properties

Wolf and McCarthy (*19*), Taylor and Kirchgessner (*17*), and Browning and Sebastian (*4*) have all used experimental data for viscosity and have extrapolated the results beyond the highest temperature for which the experimental data is available (1,980°R.). For the values of thermal conductivity the extrapolation was for temperatures greater than 1,080°R. Since the thermal conductivity shows a slightly different temperature dependence, these

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* A loan copy is available on request to Motion Picture Division, The Ohio State University, 1885 Neil Avenue, Columbus 10, Ohio. Request Reel WP (1961) Plasma Generator-Chemical Engineering (handling charge is $3.00).

authors used a Prandtl number ($c_p\mu/k$), which increased with increasing temperature. At 400°R. the value for this number is 0.68, at 2,500°R. the value is about 0.81, and extrapolated all the way out to 6,000°R. the value is 0.93. Variation of the Prandtl number does not seem justified for the following reasons.

1. In the kinetic theory of gases the same collision integral appears in both the viscosity and thermal conductivity equations. Since the temperature dependence is the same in these equations, the thermal conductivity is directly proportional to the viscosity; consequently the ratio of the two should be a constant with temperature, and if the heat capacity can be considered as a constant, the Prandtl number would remain constant with temperature. For helium one predicts 2/3, which is its measured value at normal temperatures.

2. Helium at atmospheric pressure and elevated temperatures should be closer to the model on which the kinetic theory calculations are based than it is at normal temperatures; consequently it should obey the theory better at elevated temperatures. Since it already appears to obey the model at normal temperatures, it should continue to do so at higher temperatures, as long as complications of ionization and variation of heat capacity do not enter.

3. The reduced temperature range of the present work is conservatively estimated as 21 to 640 and the reduced pressure as 0.1 to 0.5. Over this range helium is an ideal gas.

4. At the maximum temperature of this investigation (15,000°R.) helium would have an equilibrium ionization of only 0.003% in accordance with the Saha (*13*) equation. This low value is directly due to helium's high ionization potential of 24.46 ev. Also a calculation of the electronic contribution to the heat capacity for this same temperature shows that the heat capacity could be assumed constant at 1.242 B.t.u./lb.-°R.

5. The experimental work of Stroom, Ibele, and Irvine (*15*) and the comments of Brokaw (*3*) confirm these suggestions. The former authors measured the Prandtl numbers from recovery factors and obtained constant values even where one would predict a variation from the experimentally measured thermal conductivities and viscosities. The latter author suggests that less errors will be introduced if extrapolation of the collision integral is made rather than extrapolation of experimental data. Amdur (*2*) has indicated that the Prandtl number should be constant to the first approximation and that higher approximations will certainly contribute less than 0.5% as long as helium is in its normal ground state under ideal gas conditions.

For a given temperature the viscosity may be calculated from the kinetic theory as developed by Chapman and Enskog (*6*), provided the Leonard-Jones potentials are known. For a simple monatomic gas such as helium these potentials are known up to 7,400°R. Above this temperature one may use an empirical relationship which was determined to be

$$\mu = (8.50 \times 10^{-4})\, T^{0.643} \qquad (1)$$

where the viscosity is in pounds per foot-hour and the temperature in degrees Rankine. This relationship checked that predicted by kinetic theory to within 1%. Kinetic theory in turn checked the actual data at 1,980°R. to within 4%. In the calculations reported in reference *18* this procedure was used to establish the viscosity at all temperatures; however in the temperature range considered here there is up to 50% disagreement between the extrapolation and the molecular beam measurements of Amdur and Mason (*1*). The molecular beam results should be in error by about 10% at the worst and are more probably in error by only 5% over the temperature range being considered. Consequently the data in the present paper have been recalculated to reflect these better values of the transport properties.

For the determination of the thermal conductivity the Prandtl number was taken as constant and the conductivity calculated accordingly. Essentially this involved the equation

$$k = (15/4)(R/M)\,\mu = 1.859\mu \qquad (2)$$

As already indicated higher approximations would be expected to contribute less than 0.5%.

In order to calculate a representative viscosity one must assume that the temperature is isentropic; this is far from the case. With the wall temperature approximately constant at a temperature never more than slightly above the temperature of boiling water, the temperature differences in the gas are extreme. These differences varied as much as 15,000°R. in the radial direction and by as much as 10,000°R. in the axial direction. The problem can be attacked by assuming that some temperature exists that will adequately represent conditions within the tube. The bulk temperature (one-half the sum of the average inlet gas temperature and the average outlet gas temperature) was assumed to be this temperature.

Temperature

Direct measurements of the temperature is quite difficult and requires costly instrumentation. Average temperatures are considered to be adequate and can be calculated from an overall energy balance equation. The proper form of the equation under the conditions of the experiment is

$$Q = wc_p(T_2 - T_1) + \frac{w}{J}\left(\frac{v_2^2 - v_1^2}{\alpha\, g_c}\right) \qquad (3)$$

α will be 1 if the flow is laminar and about 2 if the flow is turbulent. The

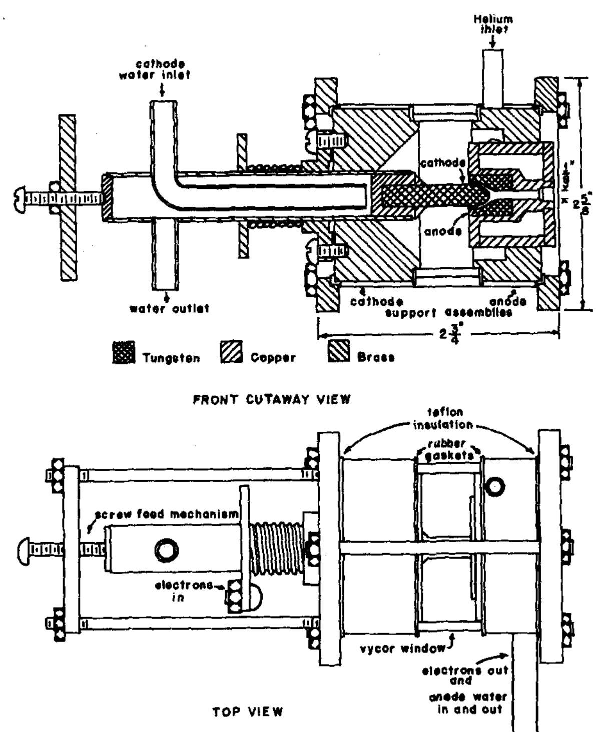

Fig. 1. Plasma generator.

TABLE 1. PER CENT DEVIATION OF ACTUAL PRESSURE DROP FROM THE LAMINAR FLOW BY THE LANGHAAR METHOD

Flow rate, lb./hr.	Temperature 2,760°R.				Temperature 15,060°R.			
	N_{Re}	L/D 11	L/D 19	L/D 27	N_{Re}	L/D 11	L/D 19	L/D 27
0.213	190	0.4	0	0	64	0	0	0
0.496	430	3	1	0.3	150	0	0	0
0.925	810	9	3.5	1.5	280	1.5	0	0

The maximum entrance effect would be about 10% and in nearly all cases less than 5%.

mechanical equivalent of heat J is included.

As the equation shows, the decrease in temperature is a function of both the heat transferred from the gas to the cooling water and the decrease in the velocity of the gas. Since it is known that helium obeys the perfect gas laws under the experimental conditions, and that the product of the velocity and density are constant for a given mass flow rate, the equation can be transformed to a function of temperature only. With the additional assumption that the pressure is nearly constant along the tube, the equation becomes

$$Q = wc_p\,(T_2 - T_1) + \left(\frac{wJ}{\alpha g_c}\right)\left(\frac{GR}{pM}\right)^2 (T_2^2 - T_1^2) \quad (4)$$

Using this relationship, the heat transferred, and the outlet gas temperature one can calculate the gas inlet temperature, provided ionization of the gas is sufficiently small that the heat capacity remains essentially constant. The gas outlet temperature was brought to a known and constant value by adding a fourth heat exchanger. The maximum temperature so calculated was approximately 15,000°R. Cann and Ducati (5) have shown that 5 pipe diam. (used here) should be long enough for thermal equilibrium to be established. Thus equilibrium values from the Saha equation should be valid and show that ionization does not contribute appreciably at this temperature level.

With an inlet temperature of 15,000°R. and a maximum axial temperature difference of 10,000°R., the change in kinetic energy of the gas (velocity head) contributed a maximum of 15% of the total heat transferred and in most cases would be much less than this. The kinetic energy term was therefore neglected in calculating temperatures.

Energy Losses

The addition of the fourth heat exchanger also allowed a heat balance over the whole system. The total heat into the arc (from the current flow through the arc and the voltage drop across the arc) and the heat given up to the cooling water were known; the difference was the heat lost either by direct radiation from the gas or thermal radiation from physical parts which had been heated by the arc. Since radiation from a monatomic gas is negligible until ionization becomes significant, essentially all heat lost by radiation was lost from the heated parts.

This radiation loss varied according to the type of arc, approximately 2,500 B.t.u./hr. for the low current arc and 1,700 B.t.u./hr. for the high current or contracted and winding arcs. The higher loss at the lower current appears due to the more diffuse nature of the arc, wherein a greater surface area of the cathode is heated to high temperatures.

Once the loss to radiation had been determined, the exit temperature of the gas could be calculated directly without the fourth exchanger. The difference between the heat input and the sum of the heat lost to the cooling water and radiation would be that carried out with the exiting gas.

Sonic Velocity

One other problem which would complicate the analysis further would be the attainment of sonic velocities in the flow tube. Based on the velocity of sound in helium at operating temperatures the Mach number calculated was never more than 0.41, even though gas velocities in excess of 2,500 mi./hr. were reached.

Entrance Effects and Flow Regime

Even though a number of rather far reaching assumptions have been involved, the Reynolds number of helium flow seems susceptible to calculation up to temperatures of 15,000°R. Thus one can determine whether the flow is laminar, provided entrance effects do not cause serious instabilities. The method of Langhaar (*10, 11*) can be used for the estimation of the pressure loss occurring during the development of laminar flow in the entrance of a pipe from a flat velocity profile. Although the geometries are not exactly the same, this method can be used to make a rough estimate of the entrance effect. Table 1 shows several values of the per cent deviation from fully developed laminar flow at the elevated temperature conditions. The error increases at room temperature to a maximum of 30% for the exchanger used in this paper. Details of the analysis are reported in reference *18*.

In trying to establish that the flow was laminar the variation of the Reynolds number along the tube length had to be considered. For the same flow conditions L/D lengths of 11, 19, and 27 were used. In twenty-seven

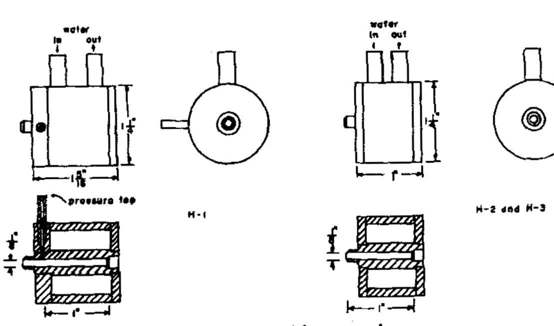

Fig. 2. Schematic of heat exchangers.

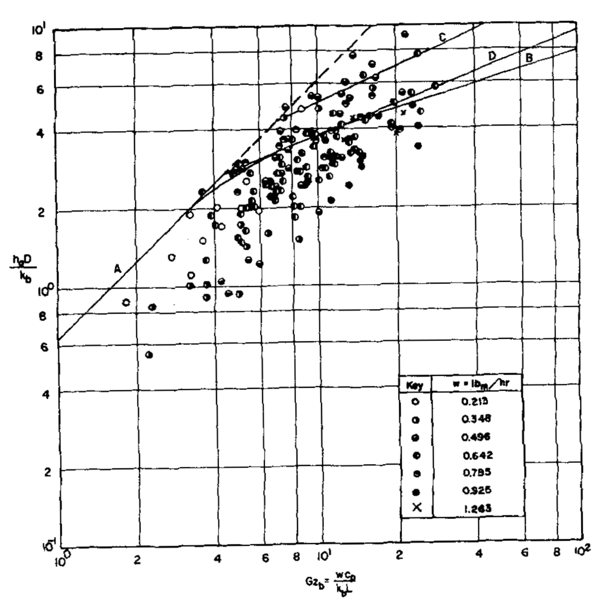

Fig. 3. Heat transfer correlations.

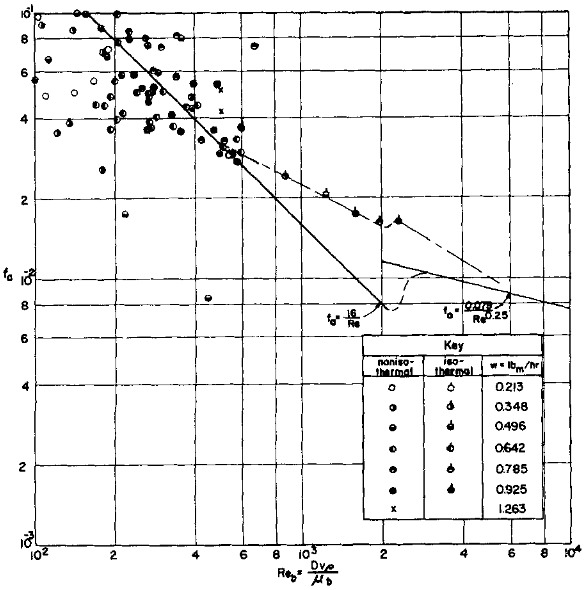

Fig. 4. Momentum transfer correlation.

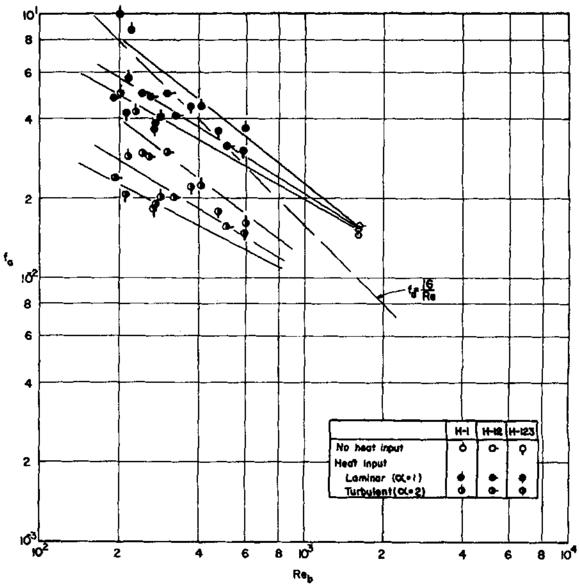

Fig. 5. Friction factor vs. Reynolds number (0.642 lb./hr.).

runs, where all three lengths were used, the deviation from the average Reynolds number over the entire length varied from 0 to 18% with most runs varying about 10%. This low variation of Reynolds number was caused by the short length of the exchange system (incomplete cooling). Even if the helium had been completely cooled by the time it left the third exchange, only in five of the one hundred fifty-nine data points would the Reynolds number exceed 2,100 at the exit. Under conditions with heat input only one run had an average Reynolds number in excess of 600 and this was a run with only one exchanger section attached. Thus based on the Reynolds number the flow should be close to laminar, and based on the entrance calculation should be fully developed. However this does not mean the velocity profile will be parabolic or even constant, since heat transfer is occurring.

RESULTS AND COMMENTS

Heat Transfer

From the heat transferred to the cooling water and the bulk temperature of the gas one could determine the heat transfer coefficient from the heat transfer equation:

$$Q = h_a A\ (T_b - T_w) \qquad (5)$$

Since the flow was shown to be laminar, the heat transfer data could be plotted according to the method of Graetz (Nusselt number $h_a D/k$, against the Graetz number wc_p/kL). It is interesting to note that the heat transfer rates from the gas were as high as 2,500,000 B.t.u./hr.-sq. ft. and average heat transfer coefficients as high as 310 B.t.u./hr.-sq. ft.-R°. This latter figure is much higher than would normally be expected in laminar heat transfer from gases, and yet still somewhat less than predicted for these extreme conditions by the Graetz solution.

The heat transfer data are plotted in Figure 3. Line *A* (Graetz numbers less than 4) is the Graetz solution for the case where the fluid outlet temperature is equal to the wall temperature; line *B* is the Graetz solution for parabolic flow when the fluid exit temperature is higher than the wall temperature; line *C* is the Drew solution for rodlike flow; and line *D* is the line recommended from the data of Cholette and Kroll.

It is interesting to note that line *D* falls above the Graetz solution. Since the data were from the heating of air by steam, the velocity profile would have been somewhat elongated from the normal parabolic shape. The line would be expected to fall below that of Graetz (parabolic velocity distribution). The answer probably lies in the simplifying assumptions required in the Graetz solution.

On the basis of velocity distribution the present data would be expected to fall between the Graetz and Drew solutions, since the velocity distribution is somewhat rodlike. Some of the data do fall in this area, but many lie outside it. When one considers the assumptions required in obtaining the temperatures, the agreement is quite good. A general curvature seems to exist as predicted by the mathematical solutions. Nearly all points lie below line *A*, which is a theoretical limit, since the gas temperature can never be lower than the wall temperature.

The deviation can be reduced by the introduction of an empirical temperature factor, that is by using an equation of the form

$$N_{Nu} = N_{Gr}\ (T_b/T_w)^{-0.4} \qquad N_{Gr} < 8 \qquad (6)$$

$$N_{Nu} = 3\ N_{Gr}^{0.45}\ (T_b/T_w)^{-0.4} \qquad N_{Gr} > 8$$

or

$$N_{Nu} = 0.049\ N_{Re}\ N_{Pr}^{1/3}\ (L/D)^{-1/2} \qquad (7)$$

However in both cases considerable scatter still exists, and neither empirical approach seems justified until improvements in equipment can be made so that gas temperatures may be better estimated. In Equation (7) the Prandtl number variation has not been checked because only one Prandtl number was used (the 1/3 power is suggested by experiments of others); in addition no temperature dependence could be detected.

The present investigation is a step towards confirming the continued usefulness of the Graetz solution even at extreme temperature differences. This solution does not take into account viscosity or thermal conductivity gradients and does not allow for distortions of the parabolic velocity pattern, but it still appears quite adequate for the prediction of heat transfer coefficients.

Momentum Transfer

Momentum transfer was correlated with the friction factor vs. Reynolds number plot. Evaluation of the Reynolds number has already been discussed. The friction factor may be evaluated from the same overall energy balance used in calculating temperatures. With a different thermodynamic definition in its transformation its form becomes

$$\Delta p_f = \Delta p + \rho_a \left(\frac{v_1^2 - v_2^2}{\alpha\ g_c} \right) \qquad (8)$$

If one assumes that the average density of the gas can be represented by

$$\rho_a = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \tag{9}$$

makes the same assumptions as in the previous transformation of the energy equation, and uses the usual definition of the friction factor

$$f_a = \Delta p_f \frac{2g_c}{4(L/D)\rho v^2} \tag{10}$$

the final relationship for evaluating the friction factor becomes (reference *18*)

$$f_a = \left(\frac{pg_cM}{G^2JR}\right)\left(\frac{1}{(L/D)}\right)\left(\frac{\Delta p}{T_1 + T_2}\right) + \left(\frac{2}{\alpha(L/D)}\right)\left(\frac{T_1 - T_2}{T_1 + T_2}\right) \tag{11}$$

The momentum transfer data are plotted in Figure 4. Also shown are the isothermal relationship $f_a = 16/N_{Re}$, a line for turbulent flow in smooth pipes, and the isothermal (room temperature) relationship developed in the present investigation. The disagreement between the accepted isothermal relationship and that of this investigation is due to the effect of the entrance, as already noted. This general effect is approximately predicted by the methods of Langhaar and is much more pronounced at lower temperatures. At high temperatures and in an approach section of 5 pipe diam. the entrance effects should not be observable (less than 5%). The observed scatter of the nonisothermal data points is thus attributed to other causes. It is undoubtedly due in part to the use of calculated temperatures, but it is also due to an L/D effect as shown in Figure 5. Here at a single mass flow rate of 0.642 lb./hr. the friction factor has been calculated by assuming first a velocity distribution corresponding to laminar flow and second a distribution corresponding to turbulent flow. Under both assumptions the L/D effect is apparent. In addition the assumption of laminar flow appears to be consistent with the isothermal points. Even at these extreme temperature gradients the friction factor can be estimated to within about 50% accuracy through the use of the standard isothermal relationship $f_a = 16/N_{Re}$.

A fact of considerable interest in the analysis of the momentum transfer data was the occurrence of negative pressure losses as high as 0.12 lb./sq. in. gauge/in. [also observed in the momentum transfer data by Talbot (*16*) and by Skrivan (*14*)]. That is the gas was apparently flowing from a lower pressure to a higher pressure. Mathematically the phenomenon ap-

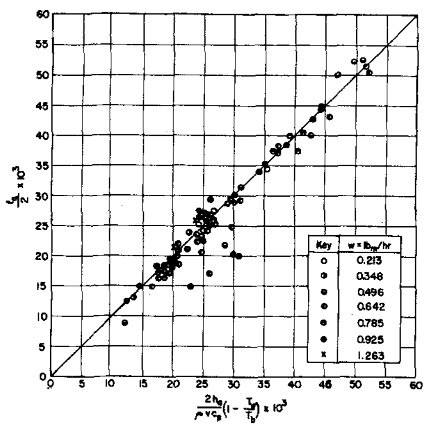

Fig. 6. Proposed analogy for heat and momentum transfer in laminar flow with gas cooling at high temperature differences.

pears readily explainable. The increase in pressure due to the loss of kinetic energy or velocity head is greater than the decrease in pressure due to frictional pressure loss. The argument does not however satisfy the reasoning that it is impossible for a flow from a lower pressure to a higher pressure to occur without some intermediate energy input. The authors propose that this too is explainable in that the momentum of the rapidly moving gas, at times in excess of an average 2,500 mi./hr., is more than sufficient to overcome the adverse pressure differential. However near the walls, where the gas has little or no momentum, the argument would not hold, and here there may be a tendency for a small annular film of gas to flow from the exit of the tube towards the entrance. Some such metastable condition was evidenced by the wide pressure fluctuations which occurred as the pressure losses tended towards positive after having been negative. Also, with the fourth heat exchanger attached, operation was always stable and nearly completely noiseless. This was probably due to the frictional pressure loss always being greater than the increase in pressure due to the kinetic energy decrease.

Analogies

Although no analogies of heat and momentum transfer for laminar flow were found in the literature, a theoretical analogy can be readily developed from the theory of molecular transport as set forth in Foust et al. (*8*). There the heat diffusivity is defined as

$$\frac{k}{\rho\, c_p} = \frac{Dh_a\, \gamma_h}{4\rho\, c_p} \tag{12}$$

and the momentum diffusivity as

$$\frac{\mu}{\rho} = \frac{Df_a\, v\gamma_m}{8} \tag{13}$$

The ratio of momentum diffusivity to heat diffusivity is the Prandtl number. Taking this ratio and rearranging one has

$$\frac{f_a}{2} = \frac{h_a}{\rho\, vc_p}\left(\frac{\gamma_h}{\gamma_m} N_{Pr}\right) \tag{14}$$

where the γ terms refer to the ratio of the difference between the average concentration of a property and its value at the wall, to the difference between the concentration of the property at the center and at the wall; that is $\gamma = (\Gamma_{avg} - \Gamma_{wall})/(\Gamma_{center\ line} - \Gamma_{wall})$.

Further manipulating Equation (11) and assuming the measured pressure loss to approach zero one obtains (reference *18*)

$$\frac{f_a}{2} = \frac{h_a}{\rho\, vc_p}(2)\left(1 - \frac{T_w}{T_b}\right) \tag{15}$$

A comparison of these two relationships suggests that, under the conditions of extremely rapid cooling of a gas in laminar flow, the following might be true:

$$\gamma_m = 1/2 \text{ or greater} \tag{16}$$

$$\gamma_h = C\left(1 - \frac{T_w}{T_b}\right) \tag{17}$$

where C is 3/2 or greater depending on γ_m. The variation in γ_m comes from the variation on the center line velocity as caused by the radial variation in density. Thus exact values are difficult to establish.

The general analogy developed for the rapid cooling rates is (reference *18*)

$$\frac{f_a}{2} = \frac{2h_a}{\alpha\rho\, vc_p}\left(1 - \frac{T_w}{T_b}\right) \tag{18}$$

which becomes

$$\frac{f_a}{2} = \frac{2h_a}{\alpha\rho\, vc_p} \tag{19}$$

when the wall temperature is small compared with the bulk temperature. For an α of 2 (corresponding to turbulent flow) this becomes the familiar Reynolds analogy. For an α of 1 (corresponding to laminar flow) one obtains a corresponding laminar analogy.

The experimental results are shown in Figure 6 for the more general form [Equation (18)]. Essentially 100% of the points fall down and to the right of the line when the temperature correction was not used. This would be for the Reynolds analogy. A marked improvement is obtained by inclusion of the derived temperature factor. The estimated gas temperature is used in both the heat transfer coefficient and friction factor calculations. Thus the improvement shown in Figure 6 could be a cancellation of errors, because the same errors appear in both terms.

It should be noted that the general analogy of Equation (18) becomes meaningless for the case of isothermal flow. But this is just one of the many anomalies that occur in considering analogies and points to the great need for more research in this area.

A final point that became apparent in this investigation stems from further consideration of Equation (11). Assuming that the pressure loss approaches zero and that the gas exit temperature is negligible compared with the inlet temperature, one obtains the following equation (reference *18*):

$$f_a = \frac{2}{\alpha(L/D)} \qquad (20)$$

This relationship explains the L/D effect observed. Using this relationship in the analogy of Equation (19) one finds a means of approximating the heat transfer coefficient for these extreme conditions; that is

$$h_a = \frac{\rho v c_p}{2(L/D)} \qquad (21)$$

Upon rearrangement this is identical to the Graetz solution for the case of outlet temperature identical to wall temperature:

$$h_a D/k = (2/\pi)(wc_p/kL) \qquad (22)$$

It is under conditions such as these that the assumption of negligible gas exit temperature should be valid.

SUMMARY

1. The Graetz type of correlation holds sufficiently well for the prediction of heat transfer coefficients for monatomic gases. However before a study of the effects of dissociation and ionization on heat transfer can be made, considerable modification of the test system will be necessary, so that the uncertainty about the gas temperature can be eliminated. Once the considerable scatter of the experimental data can be reduced to more reasonable limits, other effects and other gas systems can be investigated. A complete analysis of the various sources of error is available in reference *18*. It appears that the temperature estimation is by far the major contributor, because of insufficiently accurate voltage and current measurements and questions concerning the heat loss term. The scatter is about three times that obtained by others at much lower temperatures.
2. The isothermal relationship $f_a = 16/N_{Re}$ holds sufficiently well for engineering estimates.
3. An improved and general analogy has been formulated as Equation (18)
4. Equation (20) offers an approximate and rapid means of estimating the friction factor.
5. Equation (21) offers an approximate but rapid means of estimating the heat transfer coefficient.

ACKNOWLEDGMENT

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NOTATION

A_w = inside area of pipe, sq.ft.
c_p = specific heat (1.242 for helium), B.t.u./°R.-lb.
D = inside diameter of pipe, ft.
f = friction factor
f_a = average friction factor
g_c = dimensional constant (4.17 × 10⁸), lb.-ft./lb. f-hr.²
G = mass velocity, lb./sq.ft.-hr.
h_a = average heat transfer coefficient, B.t.u./hr.-sq.ft.-°R.
J = mechanical equivalent of heat (778), ft.-lb. f/B.t.u.
k = thermal conductivity based on T_b, B.t.u./hr.-sq.ft.-°R./ft.
L = length of pipe, ft.
M = molecular weight, lb./lb. mole
N_{Gz} = Graetz number with k evaluated at T_b (wc_p/kL)
N_{Nu} = Nusselt number based on k evaluated at T_b (h_aD/k)
N_{Pr} = Prandtl number based on k and μ evaluated at $T_b (c_p\mu/k)$
N_{Re} = Reynolds number based on μ evaluated at T_b $(Dv\rho/\mu)$
p = pressure, lb. f/sq.ft.
p_1 = pressure at entrance to exchanger, lb. f/sq. ft.
p_2 = pressure at outlet of exchanger, lb. f/sq. ft.
Δp = net pressure drop, lb. f/sq. ft.
Δp_f = pressure drop due to friction, lb. f/sq. ft.
Q = heat transfer rate, B.t.u./hr.
R = gas constant (1.985), B.t.u./lb. mole °R.
T = temperature, °R.
T_b = bulk temperature = $(T_1 + T_2)/2$, °R.
T_w = wall temperature, °R.
T_1 = temperature at inlet to exchanger, °R.
T_2 = temperature at outlet of exchanger, °R.
v = average velocity, ft./hr.
v_1 = average velocity at entrance to exchanger, ft./hr.
v_2 = average velocity at outlet of exchanger, ft./hr.
w = mass flow rate, lb./hr.

Greek Letters

α = function varying between 0 and 2 depending on velocity profile (1 for isothermal laminar flow and approximately 2 for isothermal turbulent flow)
γ_m = ratio parameter for momentum transfer
γ_h = ratio parameter for heat transfer
μ = viscosity evaluated at T_b, lb./ft. hr.
π = constant (3.1416)
ρ = density, lb./cu. ft.
ρ_1 = density at entrance to exchanger, lb./cu. ft.
ρ_2 = density at outlet of exchanger, lb./cu. ft.

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